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Sources of Difficulty in Comprehending and Solving Mathematical Word Problems

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ABSTRACT The study reported in this article sought to explore Grade 11 learners' perceptions of the sources of difficulty in comprehending mathematical word problem solving. Issues of using mathematical language, in particular the use of vocabulary knowledge in word problem solving were investigated in relation to learners' academic performances. The study discussed in this article followed a mixed-methods design. Data collection strategies included a test and a questionnaire with structured and open-ended questions. Analysis of data revealed that learners struggled with defining algebraic terms used in the word problem statements as well as in instructional vocabulary. The learner's perceptions of the sources of difficulty in comprehending and solving the mathematical word problems revealed that mathematical language impose difficult challenges to academic achievement. A correlation coefficient of r = 0.53 between vocabulary knowledge and performance in word problems suggested that knowledge of vocabulary influences success in word problem solving. In brief, the findings of the study reported here indicated that mathematical language appeared to influence learners' comprehension when solving mathematical word problems.

INTRODUCTION

The difficult part of solving mathematical word problems appears to be the process of understanding a problem and deciding what operation(s) need(s) to be performed. The study reported in this article sought to explore the issues of vocabulary and mathematical language used in mathematical word problem solving. Haliday (1978) described mathematical language as a language defined by the mathematical register. It is therefore reasonable to suggest that success in word problem solving requires a learner to gain familiarity with the vocabulary of Mathematics as defined by the register (Moschkovich 2002). Other researchers have agreed that the function of mathematical language is to help in constructing, expressing and communicating mathematical ideas and meanings during problem solving (Hans and Ginsburg 2001; Pimm 1981). Boulet (2007) further asserts that the totality of a mathematical message is often embedded in the context of a three-way relationship involving mathematical words, symbols and numerals, and that these three components define the mathematical language. It is against this background that this article reports on the influence of mathematical language and vocabulary on learners' ability to solve mathematical word problems based on linear equations.

Several studies have been carried out to investigate the effect of language on learner achievement levels in the area of mathematical modelling of word problems (Ellerton and Clarkson 1996; Pimm 1997; Fasi 1999; Moschkovich 2002; Adams 2003; Latu 2004; Garegae 2007; Schleppegrell 2007; Sepeng 2013). In their discussions, they perceive mathematical language as a language different from plain English. These studies reported that achievement in Mathematics is founded upon a strong grasp of mathematical language. The next paragraph outlines the structure of this chapter.

Nature of Mathematical Language

Mathematical language is defined by a mathematics register (Halliday 1978). Halliday views a mathematics register as a set of meanings that is appropriate to particular function of a language, together with words and structures which express these meanings. Although mathematical language is a language, it appears to be different from other languages such as French, Setswana, English, and as a results, it cannot be treated as a language on its own (Halliday 1978).

They further assert that mathematics is not a natural language as no one uses it as a first language. They conclude by saying that the function of mathematical language should therefore, be to help to express mathematical ideas and meanings in mathematical activities (Han and Ginsburg 2001).

For the purposes of the study reported in this article, mathematical language that is used here focuses on mathematical vocabulary, which includes technical terms, symbols, non-technical terms and words with multiple meanings. These components of mathematical language pose numerous challenges to learners' comprehension in solving related tasks, especially those involving word problems (Adams 2003; Schleppegrell 2007). Pimm (1981) suggested that most learners' difficulties with mathematics may be more likely due to the complexity of wording associated with written material than the nature of mathematical tasks being posed or explained.

Dale and Cuevas (1987) argued that apart from teaching learners technical vocabulary as stand-alone words teachers must encourage learners to learn definitions of technical terms within particular mathematical contexts. These scholars also argue further that with such practice learners are placed at a vantage point when it comes to word identification leading to the selection of suitable strategies and algorithms when approaching problem solving in word problems.

Factors that Influence Performance in Word Problem Solving

Mathematical problem solving is one of the most central aspects of Mathematics (Kilpatrick et al. 2001). These researchers argued that problem solving helps learners to make sense of how Mathematics can be of use in daily situations. However, some of the studies in South Africa have highlighted learners' tendencies to relegate common-sense knowledge in their reasoning and solution processes during word problem solving (for example, see Sepeng and Webb 2012). Other studies have defined problem solving as the ability to read, process, and solve mathematical situations (see Goldberg 2003). However, this ability is not nurtured by teachers as they have a tendency of encouraging mechanical way of solving word problems (Sepeng and Webb 2012). This tendency results in learners failing to develop their own personal connections and understanding between mathematical concepts (Goldberg 2003). O'Connell (2000) identified four variables that may prevent learners from entering into mathematical meaning that makes space to solve mathematical word problems, namely, culture, choice, attitude, and previous mathematical experience. The author defines mathematical meaning-making as the ability to construct meaning of mathematical experiences. According to O'Connell (2000), culture affects this space because of mediating the unfamiliar words that are found in mathematical word problems.

Research has demonstrated that learners' previous mathematical experience is influenced by the culture of the classroom (Schoenfield, 1998). Schoenfield (1998) noted that if the culture mainly focuses on memorization of steps in solving word problems, then learners might be prevented to enter this meaning-making space in solving word problems. Kilpatrick et al. (2001) argue that the attitude of learners is paramount in enabling them to enter this space. They further argue that choice is determined by learners' attitudes towards mathematics.

Vocabulary Knowledge in Mathematics Classrooms

A solid body of literature emphasised the need for learners to be conversant with the Mathematics register, especially vocabulary terms (Adams 2003; Boulet 2007; Dawe 1983; Garegae 2007; Latu 2004). These researchers called for mathematics teaching that equips learners with explicit knowledge of mathematical vocabulary through rigorous instruction programmes of these specialised terms. Kotsopoulos (2007) found that most learners experience interference when words borrowed from everyday language are used in Mathematics where they are supposed to attach different meanings depending on the context in which these words are used. The challenge on learners would then be to relearn these familiar words and assimilate them into correct mathematical contexts. One key finding from Kotsopoulos' study was that the learners' knowledge of mathematical vocabulary was paramount to their understanding of mathematical concepts, which are often introduced using these terms.

In a study that investigated the effect of using specific vocabulary in teaching junior high school mathematics learners, Dresher (1934) concluded that the teaching of specific vocabulary words before administering tests on word problems was effective. The study found that learners who had received special instruction on specific vocabulary performed better on word problems than those who were subjected to normal teaching. A key finding in Dresher study illustrated that knowledge and proficiency in mathematical vocabulary seem to increase academic achievement in word problem solving. Almost three decades later, Van der Linde (1964) carried a research where an experimental class was taught a list of mathematical vocabulary terms for 24 weeks. The results obtained from this experimental class were consistent with Dresher's (1934) findings in which greater gains were realised in both the understanding of mathematical concepts used and improved learners' problem solving abilities. These key findings reveal the role played by a comprehensive grasp of mathematical vocabulary by learners after being taught as they go on to achieve better in word problems.

Theoretical Perspectives

This study is framed within a Piaget's (1967) constructivist theory. According to Piaget, constructivism is a cognitive learning theory with a distinct focus on the mental processes that construct meaning. The general principles of constructivism are based largely on Piaget's (1967) processes of assimilation and accommodation. The two processes are mechanisms through which learning takes place.

The basic principles of constructivism propose in theory that growth in learning requires structural change and this change is brought about when a child finds aspects in the environment that are incompatible with their present cognitive schemas. Learners may be overwhelmed by newly introduced scientific concepts that are characteristic of meta-level learning (Sfard 2006). In such situations contradictions, misconceptions, perturbations and surprises lead to cognitive conflict that eventually lead to restructuring of schemas through the processes of assimilation and accommodation.

It should be noted that it is through the mechanism of equilibration that enables the child to eliminate these contradictions; incompatibilities and conflicts. Piaget (1977) argued that during the process of learning, there exists a continual search for better equilibrium as each state of equilibrium is short-lived. Meta-level learning is both characterized by meta-discursive and meta-rules that are different to those found in object-level learning. Hence, there is a need to adapt and assimilate these new mathematical objects. In the same way mathematics learning is viewed as a gradual process of learning to master the mathematics register as learners progress through grade levels (Moschkovich 2002).

From a constructivist's perspective, knowledge cannot be transferred, being intact, from one individual to the other in ready-made fashion, but instead, the child is seen as an active participant in the acquisition of new knowledge. The child is responsible for the construction of his knowledge through the interaction of new ideas with his prior knowledge. A cognitive schema therefore functions as an intellectual tool during the process of learning where a child can retrieve and apply it at appropriate times during the learning process

METHODOLOGY

The study reported here followed a mixed-methods design. The design was employed mainly because of its strength which lies in the fact that advantages of each approach complements those of the other making a stronger research design that yields more valid and reliable findings (Opie 2004). This concept of using more than one research method is referred to as triangulation (Cohen and Manion 2007). Qualitative data were collected through a questionnaire, while quantitative data were collected through a test. A test, consisting of word problems, was administered to gain understanding on the extent to which learners are able to use mathematical vocabulary during a word problem solving activity. A questionnaire, with structured and open-ended questions, was administered to explore the extent to which learners made sense of the mathematical language that were used in the word problem solving tasks.

Participants

The sample of the study consisted of 60 Grade 11 learners doing Mathematics in a township high school. Of the 60 learners who participated in the study, 29 were female and 31 were male. These learners were purposively selected based on their ability in Mathematics, Science and English at the end of their 9th Grade. In other words, we attempted to have all participating learners to enter the research process almost at the same level of academic achievement in mathematics. All the participating learners used English (a foreign/second language that is not spoken at home) as a language of learning and teaching mathematics in multilingual mathematics settings.

Research Design

As mentioned earlier, the study reported in this article followed a mixed-methods design, with qualitative data (gathered via a questionnaire) informing quantitative data (collected from a test). A test consisting of word problems was administered to Grade 11 learners. The test was used to understand how mathematical vocabulary is used by the learners during word problem solving. Qualitative data were gathered via a questionnaire with the aim of exploring how learners made sense of the mathematical language used in the word problems. Both Garegae's (2007) and Latu's (2004) analytic frameworks were adopted and used for the purposes of data analysis in this study. The study reported here sought to answer the following questions:

1. To what extent does mathematical language in word problems affect learners in producing correct process solutions?
2. What are the learners' perceptions of the sources of difficulty in comprehending and solving the mathematical word problems?

Procedure

A test was administered to all the participants in both schools at the same time of the same day. The researchers monitored invigilation sessions by availing ourselves to classrooms that participated in the study. Learners' written works were marked using a marking guideline. Achievement grading levels from 1 to 7 were used to place learners in their correct performance levels. The procedure of allocating grading levels was adapted from the South African Department of Education (DoE) to suit the contexts of a study reported in this article (DoE 2003).

Material

The following word problems (WP) tasks were given to the participating learners in a form of a test:

WP1: If the product of 2and 4 is subtracted from twice a certain number and then increased by 4, the result is 22. Find the number.

WP2: The sum of 3 consecutive numbers is 75. Find:

a. the three numbers.
b. the difference between the largest and smallest number.

WP3: A mother is three times as old as her daughter. In 12 years' time she will be twice as old as her daughter. How old is the mother now?

Learners were asked to choose a letter in order to formulate an equation in order to solve the problems. Spaces were provided to allow the learners to show all their workings and then provide a detailed written explanation of how they arrived at a particular solution. Learners' work (or scripts) where marked and coded accordingly for purposes of data analyses.

Data Analysis

Both Garegae's (2007) and Latu's (2004) analytic frameworks were adopted and used for the purposes of data analysis in this study. Learners were divided into 4 groups that were categorised according to their academic performances in both word problem solving and achievement in mathematical vocabulary. This division of learners was done to have an informed insight on learners' perceptions on vocabulary knowledge and word problems. The following criteria were used to divide learners into three groups as noted earlier:

- *Group 1* consisted of learners who highly achieved in both mathematical vocabulary and word problems.
- *Group 2* involved learners who achieved high grading scores in vocabulary test and lowly in word problems.
- *Group 3* comprised of leaners that did poorly in mathematical vocabulary but highly in word problems.
- *Group 4* had learners who performed dismally in both vocabulary and word problems.

The rationale for this categorization of learners was meant to have a representative sample of all learners' views in the study. To keep track of the selected learners and their responses, the researchers used a letter and numeral coding to indicate participants' responses. The first letter denotes the gender (male or female) and the numeral indicates the number allocated to the selected participant.

RESULTS

Qualitative Data

Results from the questionnaire were used as a lens to infer and support findings from the vocabulary and word problem test data. Qualitative data were used to answer the following research question:

What are the learners' perceptions of the sources of difficulty in comprehending and solving the mathematical word problems?

All learners in group one agreed that vocabulary knowledge in mathematics appeared to be critical component in answering questions involving word problems. Majority of these learners struggled to formulate the correct equations that were needed to answer the three WP tasks given in a test. They pointed out that although words like *product*, *increased* and *consecutive* sounded familiar to them, they could not provide their mathematical meaning when used within multiple contexts in a problem statement. The following extracts are used as examples of learners' perceptions on word problems.

F1: I struggled in this test because I did not understand the words they were talking about. I did not have an idea that I was supposed to let the unknown quantity be x or any other letter. Solving linear equations is not difficult for me but then making sense of phrases like twice a certain number confused me a lot.

It seems from F1 utterances that the mathematical language used in the word problem one (WP1), which is partly linked to the language used for teaching and learning mathematics, became an obstacle for the learners to make sense of the problem statement. This was in contrast to claims made by these learners that *solving linear equations is not difficult* to them. Majority of the learners could not realise that the phrase *this certain number* referred to the part of the equation that needed to be solved. This statement illustrates the difficulties learners faced in WP1.

F2: In question one I forgot that when a number is increased by 4, you are supposed to add 4 to it. The language is so confusing. In question two I used trial and error method to get the three required numbers, 24, 25 and 26 although I did not understand what consecutive meant.

It seems the term *increased by 4* confused this learner as she could not translate its meaning to addition. It goes on to illustrate the importance of mastering mathematical vocabulary in a problem context. It should be noted that the term *increased* is often used in ordinary English. On instructional vocabulary one learner had this to say:

F3: It was in this test that I realized that I do not actually know the difference between 'simplify, expand, solve and factorize'. I thought these words meant the same thing. I confused the term simplify from that of expand and factorize.

F3 explanation seems to suggest that knowledge of instructional vocabulary appears to be important in learners' abilities to making sense of problem statement in word problem solving. In other words, learners need to understand the terms used in word problems for them to achieve in Mathematics examinations.

As noted earlier, Group 2 was comprised of learners with high marks in vocabulary test and low marks in word problems. Most of the comments from this group of learners put emphasis around the notion of formulating (linear) equations from word problems. The learners highlighted the order of words (or terms) used in W3 task as having confused them so much that they struggled to make an attempt whatsoever in solving this problem. One of the learners argued as follows:

M4: I answered it very badly because of the order of words that were used. The words….in 12 years' time and she will be twice as old….left me confused. I didn't realise that both of them will be growing in these 12 years.

Much of the confusion was with these *relational statements* that required learners to fully comprehend how to formulate the correct equation in order to succeed in solving the task. Most of the learners could not add 12 years to the daughter's age in the second part of the question. In addition, learners in this group also encountered problems in interpreting instructional vocabulary. They could not distinguish the meaning of *simplify* from either *expand* or *factorise*.

There were very few learners who met the criteria of group 3 which was composed of learners who achieved high scores in word problem solving but poorly in vocabulary test. The learners in this category claimed that vocabulary knowledge was important for one to do well in word problems. However, they did not have a problem in understanding what the three questions required of them regarding solving the WP tasks. They indicated that only WP3 challenged them cognitively on the part that stated … *in 12 years' time she will*…. On general mathematical vocabulary, they admitted that there were some words that they were meeting for the first time such as *coefficient*, *roots of equations*, *solution* and *consecutive*. This is also reflected in Table 1 where a high percentage of learners could not define these words.

The comments from the last group demonstrated that learners lacked both vocabulary knowledge and conceptual knowledge (or understanding) to form linear equations. Their comments and the way they answered the test showed that they did not read and understand the instructions at the beginning of each section. One learner in the group had this to say:

M5: I do not understand the meaning of the words in section B. I don't understand what these word problems are saying especially when there are no numbers in them like task WP3. All questions were difficult for me especially question three.

These are generally the sentiments expressed by those learners in group four. Most of these learners did not answer all the questions in the test. All the learners in this group indicated the importance of vocabulary knowledge although they did not elaborate why it was important. They also acknowledged that they were not familiar with most of the words in the test. The learners' comments on the difficulties they faced are in line with what literature reported with regard to the possible challenges that vocabulary knowledge place on learners when they solve word problems (see for example Sepeng 2010, 2013). Learners' comments further illustrated the challenges that second language English learners face in mathematics classrooms as they struggle to conceptualise mathematical ideas. These comments support the finding that the problematic words and phrases seem to complicate learners' understanding in word problem solving (Verschaffel et al. 2000).

Quantitative Data

Mathematical Vocabulary

This section reports on the performance by learners on providing correct definitions on 15 specialized mathematical words and phrases found in Further Education and Training phase. Most of the words are linked to the vocabulary terms that are associated with the topic on Equations. For purposes of data analysis and making sense of the data in Table 1, the researchers used Garegae's (2007) analytic framework and adapted a coding system as follow: there are three sub-columns indicating three possible responses from learners namely:

- *Correct* means a learner has correctly described or defined a term or phrase using mathematical language,
- *Confused* included all cases where a learner wrote a meaningless definition of a term showing no understanding of its mathematical meaning, and
- *Blank* refers to all situations where a learner did not provide a definition of a term.

Table 1: Learners' performances on a specialised vocabulary test

List of words	*Definitions of mathematical terms as used in mathematics*		
	Column A_1 Correct (%)	*Column B_1 Confused (%)*	*Column C_1 Blank (%)*
Less than	45	54	1
Twice as old	40	42	18
Product	37	60	3
Solution	33	67	0
difference	22	75	3
Variable	33	52	15
Lowest Common Denominator	13	87	0
Consecutive	25	43	32
Linear equation	27	68	5
Sum	50	47	3
Coefficient	32	45	23
Quadratic equation	5	92	3
Factor	10	80	10
Roots of equations	3	67	30
Unknown	48	47	5
Mean	28.2	61.7	
Standard deviation	14.6	15.9	

Table 1 shows data on the performances of learners on a test. The results demonstrated that *sum* was the most correctly defined word in the test with half (or 50%) of the participating learners. It was followed by the term *unknown* (48%) and the third highest was the phrase *less than* with 45% of the learners providing correct solution responses. However, the *roots of equations* (3%), *quadratic equation* (5%), *factor* (10%) and *lowest common denominator* (13%) were found to be the most difficult words and concepts from the list. Moreover, it was interesting to note that the words *product* (37%), *difference* (22%), *consecutive* (25%) and *less than* (45%) were defined correctly by a number of learners in the test. However, only 3% of the learners managed to translate the meanings of these words into formulating correct equations. The analysis of data reported in this article seems not to suggest that defining a word used in a problem statement correctly guarantees successful word problem solving. The mean and standard deviation of correct responses was 28.2 and 14.6 respectively in the test, which suggests that on average, learners were not successful in making sense of definitions of most of the words used in the problem statements in the test.

Figure 1 shows the correlation between vocabulary knowledge and performance in word problems for all the learners in the research study.

Each learner's mark from the vocabulary test and the word problem section was converted into percentage and a scatter graph was drawn. The purpose of this scatter graph was to establish whether a relationship existed between learners' vocabulary knowledge and subsequent performances in word problems. The regression equation was found to be $y = 0.4341x + 31.238$. The correlation coefficient was found to be $r = \sqrt{0.2861} = 0.53$. The correlation coefficient of 0.53 statistically confirms that a linear relation does exist between the two variables. Correlation coefficient measures the intensity or strength of a relationship between two variables. Therefore the data in Figure 1 suggests that knowledge of mathematical vocabulary appears to enhance better performance in word problems solving. Partial results from the word problem analysis reveal that mathematical language especially relational statements negatively affect learners' understanding.

DISCUSSION

These results are in line with Latu's (2004) findings where learners confused the meanings of the word *factorise* with that of *simplify*. However, in this research study, the word *solve* was well understood in a test unlike in Latu's (2004) study where the word posed problems to the learners. According to Dale and Cuevas (1987),

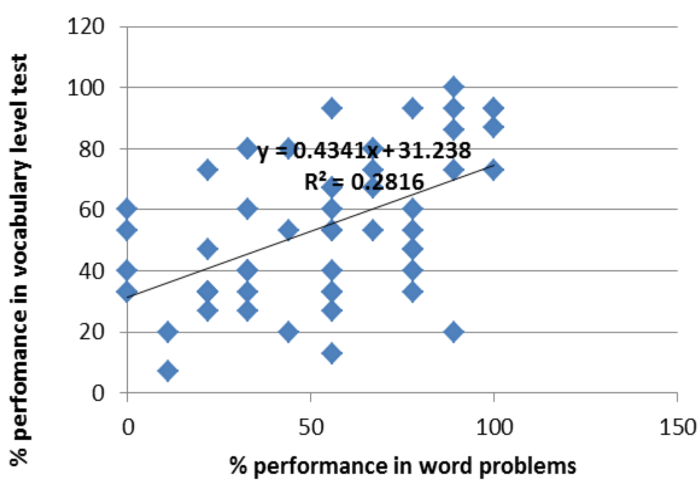

Fig. 1. Scores on vocabulary levels test vs. mathematical word problem scores [r=0.53]

as well as reports by Latu (2004), the confusion in meanings of words is evidence of restricted meanings where new vocabulary is associated with the exact contexts in which it is learnt and not the more general concept.

The data that revealed that learners had trouble in defining algebraic terms for example *roots of equations* and *quadratic equation* in a test were consistent with reports by Garegae's (2007) study of junior secondary learners. It was also of concern to our observations that although the participating learners were in Grade 11 (a highest but one Grade in a South Africa's basic education system), their performances in vocabulary knowledge were far below those of the learners in Garegae's (2007) study. This observation seems to suggest that proficiency in the language of teaching and learning may have positive gains as far as learners' performances in vocabulary knowledge is concerned. Such an observation is supported by Latu's (2004) study in which 12th year English language learners performed highly in correctly defining algebraic terms like *quadratic equation* and *roots of equations*.

The problems associated with poor vocabulary knowledge and words with *multiple meaning* may affect achievement in mathematical word problems. As a result, majority of the learners defined *solution* in the context a person having solved individual personal life problems. The other example of a problematic word in our study and those reported elsewhere in the world was *increased* as used in mathematical context (see comments by F2). The appearance of words with *multiple meanings* in mathematical texts was further suggested by Adams' (2003) study as a hindrance to learners' achievement levels in solving word problems. In her discussions, she warns educators to take note of these words as they are used by learners and to teach them in their correct mathematical situations.

CONCLUSION

The findings of this study indicate that mathematical language has a role to play in influencing learners' comprehension in the solving of mathematical word problems. The following are key findings that emerged from the research study.

- The study revealed that learners did not have sufficient grasp of vocabulary knowledge, and did not recognise the importance of possessing this knowledge. On specialised vocabulary words, the word *sum* was the only word that was defined correctly by 50% of learners;
- Learner's perceptions of the sources of difficulty in comprehending and solving the mathematical word problems revealed that mathematical language seems to impose challenges on learners' academic achievement;
- A correlation coefficient of 0.53 between vocabulary knowledge and performance in word problems suggested that knowledge of vocabulary influences success in word problem solving; and,
- The results of this study seem to suggest that vocabulary teaching in mathematics topics positively improves academic achievement in word problem solving.

From the above key findings it can be concluded that the mathematical language that was used in word problems appeared to be an academic obstacle to learners' success in word problem solving. The results of this study seem to suggest that the inability to decode meanings of words correctly in a word problem, resulted in learners failing to produce correct solutions.

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